Charge pumping and noise in a one-dimensional wire with weak electron-electron interactions

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We consider the adiabatic pumping of charge through a mesoscopic one-dimensional wire in the presence of electron-electron interactions. A model of static potential in-between two-delta potentials is used to obtain exactly the scattering matrix elements, which are renormalized by the interactions. Two periodic drives, shifted one from another, are applied at two locations of the wire in order to drive a current through it in the absence of bias. Analytical expressions are obtained for the pumped charge, current noise, and Fano factor in different regimes. This allows us to explore pumping for the whole parameter range of pumping strengths. We show that working close to a resonance is necessary to have a comfortable window of pumping amplitudes where charge quantization is close to the optimum value; a single electron charge is transferred in one cycle. Interactions can improve the situation, the charge Q is closer to one-electron charge, and noise is reduced, following a $Q(e-Q)$ behavior, reminiscent of the reduction in noise in quantum wires by $T(1-T)$, where T is the electron transmission coefficient. For large pumping amplitudes, this charge vanishes, and noise also decreases but slower than the charge.

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I. INTRODUCTION

The suggestion that electrons can be supplied one by one by a mesoscopic circuit has been proposed over two decades ago.[1] Instead of applying a constant bias voltage to the system, it is possible to supply ac gate voltages which perturb the system periodically. Under certain conditions, the charge transferred from one lead to the other, during one period, can be almost quantized. Adiabatic pumping of electrons could in principle be used in future nanoelectronic schemes based on single electron transfer, and it also has applications to quantum information physics. Over the years, theoretical approaches to this adiabatic pumping based on scattering theory have become available.[2–4] These situations typically describe mesoscopic systems which are large enough or sufficiently well connected to leads that electronic interactions (charging effects, for instance) can be discarded. Scattering theory has been applied[4] to calculate both the charge and noise in systems in the absence of electron-electron interactions.

On the experimental scene, Coulomb blockade effects have been successfully exploited to achieve pumping with isolated quantum dots.[5] To our knowledge, pumping experiments which are not entirely based on Coulomb blockade, where the shape of the electron wave functions is modified in an adiabatic drive, are rather scarce. A recent study[6] has dealt with the transport through an open quantum dot where such interactions are minimized.

Besides Coulomb blockade physics, the effect of electron-electron interactions in conductors with reduced dimensionality has been discussed by several authors.7 The case of strong interactions in a one-dimensional (1D) quantum wire was presented in Ref. 8 using Luttinger liquid theory. Alternatively, Lal *et al.*[9] discussed the opposite limit, where the effect of weak interactions can be included in a scattering formulation of pumping using renormalized transmission/reflection amplitudes.[10] However, the results for the pumped charge remain mostly numerical in this work.

The conditions under which pumping amplitude and interactions must be tuned to achieve quantized pumping are not obvious. Many physical parameters enter this problem, such as the amplitude of the pumping potentials, the phase difference between these, the possibility of a constant offset on these potentials, the overall conductance of the unperturbed structure, and to what extent the strength of electron-electron interactions play a role. Analytical results on this issues are highly desirable, as well as information about the noise.

With regard to the experiment of Ref. 6, there is clearly a need for further understanding the role of weak interactions in such mesoscopic systems in the presence of pumping. The purpose of the present work goes in this direction, in the sense that we provide analytical expressions for the pumped charge and the noise for a one-dimensional wire in the presence of interactions. This allows us to explore all pumping regimes[11] (weak to strong pumping) and to determine in which manner and to what extent the pumped charge can help us to achieve single electron transfer. Besides addressing the question of the ideal conditions for good charge quantization, we shall establish relationships between charge and noise in different regimes. For concreteness, a two-delta-potential model will be used and interactions will be added on top of it.

II. PUMPED CHARGE AND NOISE

A. Adiabatic pumping in noninteracting systems

Here, we recall the formula which was established for the charge transferred during the single period of an adiabatic pumping cycle through a quasi-one-dimensional system. The system is in general described by a potential $V(x)$ containing two internal parameters which are modulated periodically. The time dependence is assumed to be sufficiently slow so that, although the scattering matrix depends on time, its variations are minute when an electron is scattered in the mesoscopic wire.

At finite temperature, the pumped current reads[12]

$$Q = e\int_0^{2\pi/\omega} dt \int f(E)\mathrm{Tr}\{S^{\dagger}(E,t)\sigma_z S(E,t) - I\}\frac{dE}{2\pi}, \quad (1)$$

where $S(E,t)$ is the Wigner transform of the scattering matrix $S(t,t')$ and a good approximation to the scattering matrix for the problem with the potential frozen. σ_z is the usual Pauli matrix, I the identity matrix, and f the Fermi-Dirac function,

$$S(E,t) = \int_{-\infty}^{\infty} e^{-iE(t-t')}S(t,t')dt'. \quad (2)$$

The pumping potential will generate sidebands at $E\pm\hbar\omega$ and we assume that temperature is much smaller than ω, i.e., $k_BT \ll \hbar\omega$, so that we can approximate the Fermi function by a step function. In fact, temperature dependence occurs in two places in this problem. First in the Fermi function and second, the scattering matrix elements depend on the temperature because of the renormalization due to the interactions (see Sec. II B). Formulas for averaged current and zero-frequency noise can be carried out using results of the literature in the "zero-temperature" formalism, except that the scattering matrix elements are, in fact, temperature dependent.

The pumped charge reduces to a time integral over a pumping cycle,[2,9]

$$Q = \frac{e}{2\pi}\int_0^{2\pi} \mathrm{Im}\left[\left(\frac{\partial s_{11}}{\partial X}s_{11}^* + \frac{\partial s_{12}}{\partial X}s_{12}^*\right)\frac{dX}{dt} + (X\leftrightarrow Y)\right]dt, \quad (3)$$

where s_{1i} $(i=1,2)$ are the elements of the scattering matrix $s(E)$,

$$s(E) = e^{i\phi}\begin{pmatrix} -i\sqrt{R}e^{i\theta} & \sqrt{T} \\ \sqrt{T} & -i\sqrt{R}e^{-i\theta}\end{pmatrix}, \quad (4)$$

where ϕ is the phase accumulated in a transmission event and θ is the phase characterizing the asymmetry between the reflection from the left-hand side and from the right-hand side of the potential. Conservation of probabilities imposes $R+T=1$. We assume the quantities $\sqrt{R}$, $\sqrt{T}$, θ, and ϕ to be functions of the Fermi energy E_F and of the external time-varying parameters $X(t)$ and $Y(t)$.

B. Inclusion of weak interactions

In the case of weak interactions, the transmission and the reflection amplitudes s_{12} and s_{11} can be calculated in the presence of Coulomb interaction via a renormalization procedure.[10] High energy scales above a given cutoff are eliminated. The high energy cutoff is lowered progressively. The renormalization has to be stopped when the temperature becomes comparable to this cutoff. Finally, if $s_{12}^{(0)}$ and $s_{11}^{(0)}$ denote, respectively, the transmission and reflection coefficient without interactions, s_{12} and s_{11}, can be expressed in the form[10]

$$s_{12} = \frac{s_{12}^{(0)} l^{\alpha}}{\sqrt{1+T_0(l^{2\alpha}-1)}}, \quad (5)$$

$$s_{11} = \frac{s_{11}^{(0)}}{\sqrt{1+T_0(l^{2\alpha}-1)}}, \quad (6)$$

where $l=k_B\Theta/W$; Θ is the temperature, k_B the Boltzmann constant, and W the original bandwidth. α is a negative exponent related to the strength of the screened Coulomb interaction potential.[10] Specifically,

$$\alpha = \frac{V_c(2k_F) - V_c(0)}{2\pi v_F}, \quad (7)$$

with $V_c(q)$ as the Fourier transform of the screened Coulomb potential at $q=2k_F$ and $q=0$, respectively, and $V_c(0)$ is finite due to screening. $\alpha=0$ corresponds to the absence of electron-electron interactions. $T_0=|s_{12}^{(0)}|^2$ represents the conductance of the wire in units of e^2/h. From now on, Q will denote the pumped charge with interactions and Q_0 without interactions. The integrand of Eq. (3) is therefore modified by the presence of the interactions. Thus, temperature dependence occurs through the renormalization of the S matrix. We shall be interested in the regime where temperature is much lower than pumping frequency, $k_B\Theta \ll \omega$, as said before, but renormalization of S matrix should not be too severe so that the renormalization of S matrix still makes sense. At very low temperature, all barriers become almost opaque and bosonization is required,[7] so typically $l^{\alpha} > 10^{-1}$, that is, $k_B\Theta \gg W10^{1/\alpha}$. For example, for nanotubes having α around -0.3, this gives $\hbar\omega \gg k_B\Theta \gg 10^{-3}W$.

C. Two-delta-potential model and pumped charge

Consider the pumping charge Q transferred during a single period through a 1D chain of an arbitrary potential shape. Let two parameters of the system be modulated periodically. The single-particle Hamiltonian reads

$$H = \frac{\hbar^2k^2}{2m} + V(x) + V_p(x,t), \quad (8)$$

where $V_p(x,t)$ is the time-dependent perturbation part of the arbitrary potential and has δ-like potential form,

$$V_p(x,t) = 2k_F X(t)\delta(x-x_i) + 2k_F Y(t)\delta(x-x_f), \quad (9)$$

with the amplitudes $X(t)=V_i(t)/2k_F$ and $Y(t)=V_i(t)/2k_F$, where $V_i(t)$ and $V_f(t)$ have periodic time evolutions with the same period $t_0=2\pi/\omega$ and k_F denotes the Fermi wave vector. $V(x)$ is the static potential in-between two δ potentials. Below, superscript indexes (0) indicate noninteracting systems.

Using the known relation,[13]

$$s_{\alpha\beta}^{(0)} = -\delta_{\alpha\beta} + 2ik_F G^0(x_\alpha, x_\beta), \tag{10}$$

where $G^0(x_\alpha, x_\beta)$ are the usual real-space retarded Green's functions and the fact that the functional derivative of the Green's function $\delta G^0/\delta V(x)$ can be written as the product of two Green's functions,[14]

$$\frac{\delta G^0(x_\alpha, x_\beta)}{\delta V(x_j)} = G^0(x_\alpha, x_j) G^0(x_j, x_\beta). \tag{11}$$

For the first parenthesis in Eq. (3), we get

$$\frac{\partial s_{11}^{(0)}}{\partial X} s_{11}^{(0)*} + \frac{\partial s_{12}^{(0)}}{\partial X} s_{12}^{(0)*} = \frac{(1 + s_{11}^{(0)})(1 + s_{11}^{(0)*})}{2ik_F}, \tag{12}$$

where we used the condition $s_{11}^{(0)} s_{11}^{(0)*} + s_{12}^{(0)} s_{12}^{(0)*} = 1$, and $s_{12}^{(0)}$ and $s_{11}^{(0)}$ are the bare transmission and reflection amplitudes from the disordered system, without taking into account the electron-electron interactions. For the second parenthesis in Eq. (3), we get

$$\frac{\partial s_{11}^{(0)}}{\partial Y} s_{11}^{(0)*} + \frac{\partial s_{12}^{(0)}}{\partial Y} s_{12}^{(0)*} = \frac{s_{12}^{(0)}}{2ik_F}[s_{12}^{(0)} s_{11}^{(0)*} + s_{12}^{(0)*}(1 + s_{22}^{(0)})] = \frac{|T_0|^2}{2ik_F}. \tag{13}$$

Here, we used the current conservation requirement $s_{21}^{(0)} s_{11}^{(0)*} + s_{12}^{(0)*} s_{22}^{(0)} = 0$. $s_{22}^{(0)}$ is the reflection amplitude from the right of the scatterer and can be presented as

$$s_{22}^{(0)} = 2ik_F \frac{\partial \ln s_{12}^{(0)}}{\partial Y} - 1. \tag{14}$$

Substituting Eqs. (12) and (13) into Eq. (3), using identity (14), we finally arrive at

$$Q_0 = -\frac{e}{2\pi} \int_0^{2\pi/\omega} \left(\frac{\partial \ln s_{12}^{(0)}}{\partial X} \frac{\partial \ln s_{12}^{(0)*}}{\partial X} \frac{dX}{dt} + T_0 \frac{dY}{dt} \right) dt. \tag{15}$$

Similar expression for Q_0 can be found in Ref. 15.

Using the same method for Q, we obtain, in the presence of interactions,

$$Q = Q_0 - \frac{e(l^{2\alpha} - 1)}{2\pi} \int_0^{\frac{2\pi}{\omega}} \left[\left(\mathrm{Im}\left\{ \frac{\partial \ln s_{12}^{(0)}}{\partial X} \right\} - \left| \frac{\partial \ln s_{12}^{(0)}}{\partial X} \right|^2 \right) \frac{dX}{dt} + \left(\mathrm{Im}\left\{ \frac{\partial \ln s_{12}^{(0)}}{\partial Y} \right\} - T_0 \right) \frac{dY}{dt} \right] \frac{T_0}{1 + T_0(l^{2\alpha} - 1)} dt. \tag{16}$$

For numerical simulations, we specialize to the case where the static potential $V(x) = 0$ in Eq. (8) and the time-dependent perturbations are two δ-like potentials separated by a distance $2a$. The expressions of $s_{12}^{(0)}$ and $s_{11}^{(0)}$ are needed when the single-particle Hamiltonian reads

$$H = \hbar^2 k^2/2m + V_p(x,t). \tag{17}$$

The elements of the S matrix in the absence of electron-electron interactions are given by

$$s_{11}^{(0)} = \frac{\{(\bar{Y} - \bar{X})\sin(2k_F a) - i[2\bar{X}\bar{Y}\sin(2k_F a) + (\bar{X} + \bar{Y})\cos(2k_F a)]\}}{D}, \tag{18}$$

$$s_{12}^{(0)} = \frac{1}{D}, \tag{19}$$

with

$$D = \{1 - 2\bar{X}\bar{Y}\sin^2(2k_F a) + i[\bar{X} + \bar{Y} + \bar{X}\bar{Y}\sin(4k_F a)]\}, \tag{20}$$

and $\bar{X} = X/2$, same for $\bar{Y}$. $s_{21}^{(0)} = s_{12}^{(0)}$ and $s_{22}^{(0)}$ is obtained by replacing $\bar{Y}$ by $\bar{X}$ in $s_{11}^{(0)}$.

D. Noise

Avron *et al.*[15] studied adiabatic quantum pumping in the context of scattering theory. Their goal was to derive under what conditions pumping could be achieved optimally, in a noiseless manner, with the assumption that the pumping frequency is small compared to the temperature. This enabled the authors to derive expressions not only for the pumped charge per cycle but also for the pumped noise, the current-current correlation function, averaged over a time which is long compared to the period of the adiabatic drive, at zero frequency. Specifically, the noise is defined from the current-current time correlator,

$$S(t,t') = \frac{1}{2} \langle \delta I(t) \delta I(t') + \delta I(t') \delta I(t) \rangle, \tag{21}$$

with $\delta I = I - \langle I \rangle$. This correlator is then averaged over n_0 periods of the pumping drive with n_0 large, and it is taken at zero frequency by performing an integral over the remaining time argument. Setting $\tau_0 = n_0 2\pi/\omega$,

$$S(\Omega = 0) = \frac{\omega}{2\pi n_0} \int_0^{\tau_0} dt \int_{-\infty}^{\infty} dt' S(t,t'). \tag{22}$$

Moskalets and Büttiker[4] extended the results of Ref. 15 to the case where this limiting assumption is relaxed, yielding a complete description of the quantum statistical properties of an adiabatic quantum pump, albeit restricted to small pumping amplitudes. Results made use of the generalized emissivity matrix. These results were generalized to arbitrary pumping amplitudes by Polianski *et al.*[12] Here, our goal is to address the question whether electron-electron interactions affect the pumping noise and how. Following formula (14) of Ref. 12 and applying their Eq. (15) without assuming that the pumping amplitudes $X(t)$ are small, it is possible to set the zero-frequency noise S for arbitrary pumping amplitudes into the form

$$S(\Omega=0)=\frac{1}{(2\pi\hbar)^2}\frac{e^2}{\tau_0}\int_0^{\tau_0}dt\int_{-\infty}^{\infty}d(t'-t)$$
$$\times\int_{-\infty}^{\infty}f(-\epsilon_1)\int_{-\infty}^{\infty}f(\epsilon_2)$$
$$\times\mathrm{Tr}[s(\epsilon_1,t)^{\dagger}\sigma_z s(\epsilon_2,t)s(\epsilon_2,t')^{\dagger}\sigma_z s(\epsilon_1,t')-I]$$
$$\times e^{i\ (t-t')(\epsilon_1-\epsilon_2)/\hbar}d\epsilon_1 d\epsilon_2, \tag{23}$$

where $s(\epsilon_1,t)$ is the 2×2 S matrix for an incoming wave at energy $\epsilon_1+\epsilon_F$ and value of the pumping parameter $X(t)$. τ_0 is a time which is much larger than the period τ. $\epsilon_1=\hbar^2k_1^2/2m-\epsilon_F$, where k_1 is a wave vector and ϵ_F is the Fermi energy $\epsilon_F=\hbar^2k_F^2/2m$. f is the Fermi-Dirac function. Here, $s(\epsilon_1,t)$ is taken for fixed t.[12] Since we work at temperature much smaller than $\hbar\omega/k_B$, as explained before, we can set $f(-\epsilon_1)=1$ for $\epsilon_1>0$ and 0 otherwise. We set $\epsilon_2'=-\epsilon_2$ so that both ϵ_1 and ϵ_2' will be positive. M is defined as

$$M(\epsilon_1,\epsilon_2',t)=s(\epsilon_1,t)^{\dagger}\sigma_z s(-\epsilon_2',t). \tag{24}$$

S now reads

$$S=\frac{e^2}{(2\pi\hbar)^2}\lim_{\tau_0\to\infty}\int_0^{\tau_0}\frac{dt}{\tau_0}\int_{-\infty}^{\infty}dt'\int_0^{\infty}d\epsilon_1\int_0^{\infty}d\epsilon_2'$$
$$\times\mathrm{Tr}\{M(\epsilon_1,\epsilon_2',t)M(\epsilon_1,\epsilon_2',t')^{\dagger}-I\}e^{-i[(t-t')(\epsilon_1+\epsilon_2')/\hbar]}. \tag{25}$$

Now, for large pumping amplitudes, the above formula needs to be rearranged using the fact that $X(t)$ is a periodic function of period $2\pi/\omega$. Note that the dependence of M on ϵ_1 and ϵ_2' prevents the direct use of fast Fourier transform. Nevertheless, we can use the fact that, for given values of ϵ_1 and ϵ_2', $M(t)$ and $M(t')$ are periodic functions of t. Switching to Fourier transform,

$$\hat{M}_n(\epsilon_1,\epsilon_2')=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}M(\epsilon_1,\epsilon_2',t)e^{-in\omega t}dt, \tag{26}$$

$$M(\epsilon_1,\epsilon_2',t)=\sum_{n=-\infty}^{+\infty}\hat{M}_n(\epsilon_1,\epsilon_2')e^{in\omega t}. \tag{27}$$

Performing the trace, we arrive at

$$S=\frac{e^2}{2\pi\hbar^2}\int_0^{\infty}d\epsilon_1\int_0^{\infty}d\epsilon_2'\sum_{n=-\infty}^{\infty}(|\hat{M}_{1,1n}|^2+|\hat{M}_{1,2n}|^2+|\hat{M}_{2,1n}|^2$$
$$+|\hat{M}_{2,2n}|^2-2\delta_{n,0})\,\delta\!\left(\frac{\epsilon_1+\epsilon_2'}{\hbar}-n\omega\right), \tag{28}$$

where $\delta_{n,0}$ is 1 if $n=0$ and 0 otherwise and $\hat{M}_{i,jn}$ is the (i,j) element of matrix of $\hat{M}_n(\epsilon_1,\epsilon_2')$, where energy dependences have been omitted to ease the notations.

When $\hbar\omega$ is much smaller than ϵ_F, formula (28) can be simplified. In this case, $M(\epsilon_1,\epsilon_2',t)$ will be different from $M(0,0,t)$ only when ϵ_1 or ϵ_2' are a non-negligible fraction of ϵ_F. This occurs because $M(0,0,t)$ corresponds to matrix M for incident wave and outgoing wave at energy ϵ_F. We denote by ϵ_{1F} typical energies of the order ϵ_F. ϵ_{1F} will correspond to n of the order $(\epsilon_{1F}/\hbar\omega)$, which is very large. The Fourier transform $\hat{M}_n(\epsilon_1,\epsilon_2')$ will decrease exponentially with n for large n. Thus, we can neglect the dependence on ϵ_1 and ϵ_2' and replace them by zero, which amounts to replacing the energies by ϵ_F, except in the argument of the δ function. Under these conditions, we have

$$S\simeq e^2\frac{\omega}{2\pi}\left[\sum_{n\geq 1}n(|\hat{M}_{1,1n}|^2+|\hat{M}_{1,2n}|^2+|\hat{M}_{2,1n}|^2+|\hat{M}_{2,2n}|^2)\right]. \tag{29}$$

With our form of the S matrix, this formula is equivalent to Eqs. (24a)–(24c) of Ref. 16, apart from an overall factor 2 (see Appendix C for details). For numerical simulations however, we did not make this simplification and kept the dependence on ϵ_1 and ϵ_2' of Eq. (28).

III. DISCUSSION OF PHYSICAL RESULTS

We now illustrate these formulas by computing the charge and noise in the case of two-delta-potential model. The two parameters of the drive X and Y [Eq. (9)] are chosen to vary periodically according to

$$X=X_0+\eta\cos(\omega t), \tag{30}$$

$$Y=Y_0+\eta\cos(\omega t-\varphi), \tag{31}$$

where X_0 is a constant offset potential and φ a phase difference. Note that X, X_0, and η are all dimensionless [see Eq. (9)]. To ensure maximal pumping, we shall specialize[11] to $\varphi=\pi/2$.

A. Zero offset

First, the case without offset $X_0=Y_0=0$ is studied. To look at the influence of interactions, we plot in Fig. 1 the pumped charge in units of e, with interactions and without, versus the amplitude of the drive η for an interaction parameter $l^{2\alpha}=0.3$ (moderate electron-electron interactions). There are three regimes: weak pumping, $\eta\ll 1$, intermediate pumping, η of order 1, and large pumping amplitudes, $\eta\gg 1$. The current noise times $2\pi/\omega$, in the limit of small ω, is plotted together in the same figure in units of e^2. Analytically, for $\eta\ll 1$, Q reads

$$Q=\frac{e}{4}\sin(4k_Fa)l^{-2\alpha}\eta^2. \tag{32}$$

As noted in Ref. 9, in the weak pumping regime, charge Q is larger with interactions by a factor $l^{-2\alpha}$ (see Fig. 2). Results in Ref. 4 for the noise valid for weak pumping and no

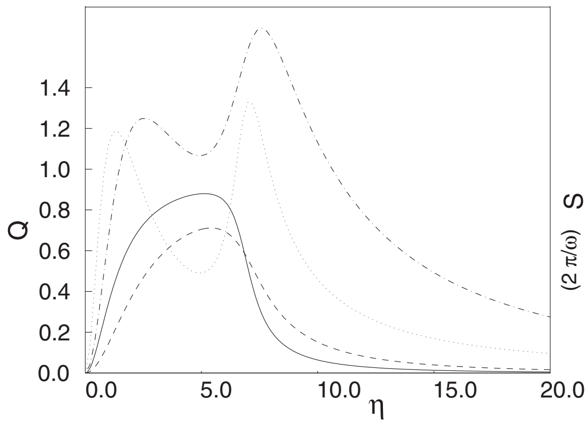

FIG. 1. Q, charge with interactions (solid line), and Q_0, charge without interactions (dashed line), both in units of e. S noise with interactions (dotted line) and S_0 without interactions (dashed dotted line), multiplied by $2\pi/\omega$, in units of e^2 vs η. Essential parameters are $X_0=0$, no offset, $l^{2\alpha}=0.3$, $k_Fa=0.5$, and $\hbar\omega=10^{-2}\epsilon_F$.

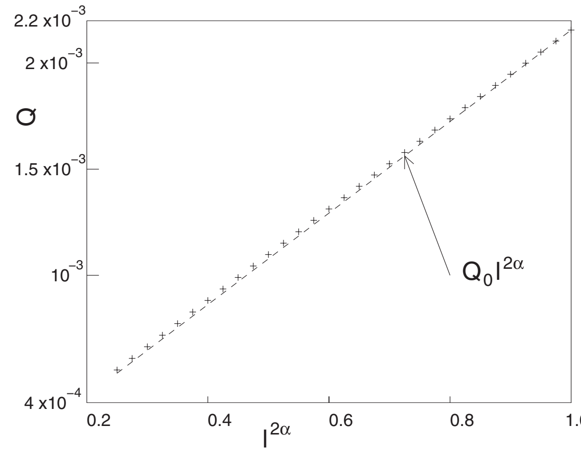

FIG. 3. Q, charge with interactions (pluses), and $Q_0l^{2\alpha}$ (lower dashed line), both in units of e vs $l^{2\alpha}$ for $\eta=15$, $X_0=0$, $k_Fa=0.5$, and $\hbar\omega=10^{-2}\epsilon_F$. In this regime, Q is approximately larger than Q_0 by a factor $l^{-2\alpha}$.

interactions can be adapted in a straightforward fashion to the case with interactions. We find the following formula for the noise for weak pumping:

$$S = e^2 l^{-2\alpha}\eta^2 \frac{\omega}{2\pi}. \tag{33}$$

The noise is thus increased by the same factor as the current. The Fano factor, defined as the ratio $S/e\langle I\rangle$, is $4/\sin(4k_Fa)$ and remains independent of the interactions, as long as we remain in the weak pumping regime. This corresponds to the very left part of Fig. 1 for η smaller than 0.25, typically.

At intermediate pumping amplitudes, Q reaches a maximum value $Q_{\max}$ which is again larger than its noninteracting analog $Q_{0\,\max}$. This maximum is of the order of the single electron charge but less than it. Meanwhile, the noise decreases. This is a reminder of the reduction in the noise by a factor $T(1-T)$, where T is the electron transmission coefficient in quantum wires. This explains why the noise exhibits a first maximum around η close to 1; since Q gets closer to one-electron charge, noise will decrease. Then, for moderate amplitudes, η around 6, charge decreases and passes through the value $0.5e$, this corresponds then to the second maximum of the noise.

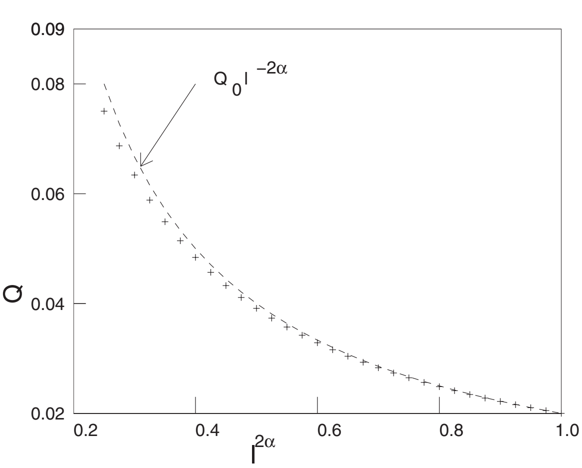

FIG. 2. Q, charge with interactions (pluses), and $Q_0l^{-2\alpha}$ (upper dashed line), both in units of e vs $l^{2\alpha}$ for $\eta=0.3$, $X_0=0$, $k_Fa=0.5$, and $\hbar\omega=10^{-2}\epsilon_F$.

For large but not very large pumping amplitudes, typically $\eta=10$, Q remains smaller than Q_0 but behaves in the same way, namely, as η^{-3}, as noted in Ref. 11. As a function of the interaction parameter, Q behaves as $Q_0l^{2\alpha}$ (see Fig. 3). For very large pumping amplitudes (typically order 100 or more), Q becomes practically equal to Q_0 and $Q-Q_0$ behaves as η^{-4} (see Appendix B for details).

For the noise, we found numerically that S and its analog without interactions, S_0, both decrease as η^{-2}, much slower than the charge (see below for analytical derivations). As concerns now the interaction dependence of the noise, S is always smaller than S_0, but for very large η, S tends toward S_0. More precisely, for large but still reasonable η, of the order 10 typically, S is almost equal to $S_0l^{2\alpha}$, whereas for very large η, of the order 100 or more, S and S_0 are practically the same. This is not surprising since Q and Q_0 are then also practically equal in the end.

This dependence on the interaction parameter is shown in Fig. 4. We have to plot $(2\pi/\omega)S\eta^2$ vs $l^{2\alpha}$, but the overall factor $2\pi/\omega$ is unimportant; the product $S\eta^2$ can be compared with both $l^{2\alpha}S_0\eta^2$ and to $S_0\eta^2$ for $\eta=15$ and $\eta=200$. For $\eta=15$, we see that η^2S is fairly well approximated by $\eta^2S_0l^{2\alpha}$. On the contrary, for $\eta=200$, such a fit fails and instead η^2S is almost equal to η^2S_0.

The results at large η can be derived from analytical formula for the charge and noise. An expansion for large η is performed, and X and Y behave as η, except at particular points where X or Y are zero (see Appendix A for details).

B. Nonzero offset

We now turn to the case where X_0 is nonzero, which enables us to have regions where Q is almost quantized. There

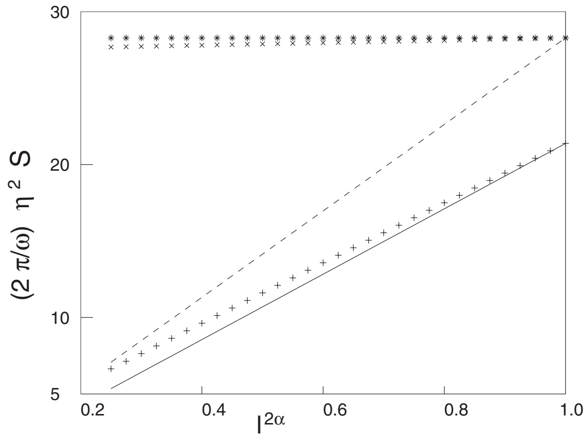

FIG. 4. $(2\pi/\omega)\eta^2 S$, noise with interactions scaled by η^2 and by the period (+), and $(2\pi/\omega)\eta^2 S_0 l^{2\alpha}$ (lower solid line) for moderate $\eta=15$. Also illustrated by the top two curves is the very large pumping amplitude regime, $(2\pi/\omega)\eta^2 S$ ($\times$) and $(2\pi/\omega)\eta^2 S_0$ ($*$), for very large η; $\eta=100$. The top two curves are close to each other. An attempt to fit $\eta^2 S$ by $l^{2\alpha}\eta^2 S_0$ (dashed line) for $\eta=100$ clearly fails for this regime of pumping amplitudes. $k_F a=0.5$ and $\hbar\omega=10^{-2}\epsilon_F$ for all cases.

are basically three cases, according to the value of $k_F a$.

The first case corresponds to $k_F a=n\pi/2$ (rigorously), where n is an integer. In this case, it is impossible to pump anything. The reason is given below. The second case corresponds to the case where $k_F a$ is small but nonzero, 0.1 typically. We first describe the behavior, then give numerical illustrations, and last provide analytical justifications. In this case, the charge is almost zero up to $\eta=X_0$. It rises quickly around $\eta=\sqrt{2}X_0$ and reaches a value close to quantized e for a wide range of values of η. This is the quantized region of η. The width of this region can be shown to scale approximately as $(k_F a)^{-1}$. After the end of this region, Q and Q_0 first

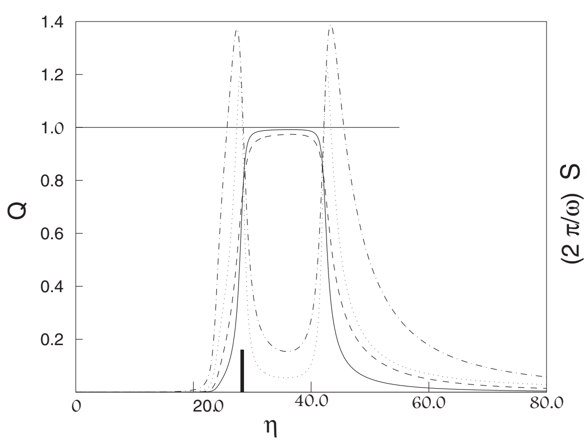

FIG. 5. Q (solid line), curve with a plateau for η between 28 and 43 (the quantized region), and Q_0 (dashed line). The two curves with two spikes located at η around 28 and at η around 43 represent $(2\pi/\omega)S$ (dotted line) and $(2\pi/\omega)S_0$ (dashed dotted line) vs η. Essential parameters are $X_0=20$, $l^{2\alpha}=0.3$, $k_F a=0.1$, and $\hbar\omega=10^{-2}\epsilon_F$. Electron charge e is set to 1. The solid horizontal line of ordinate 1 and the thick vertical line at $\eta=X_0\sqrt{2}\simeq 28.28$ are guides for the eyes.

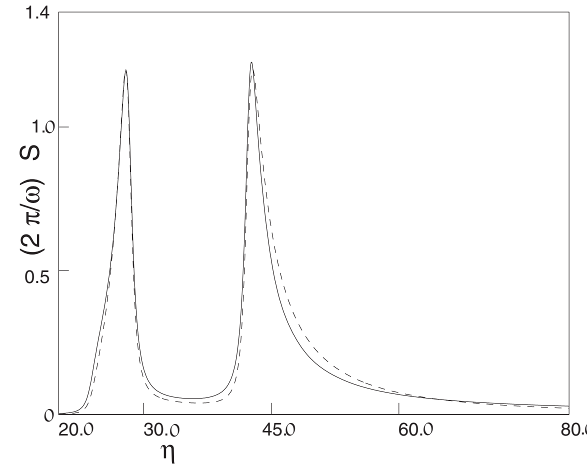

FIG. 6. S (solid line), in units of e^2, and best fit of the form $CQ(e-Q)$ (dashed line) in the interval $20\leq\eta\leq 45$ for the same situation as Fig. 5. The fit no longer works in the large pumping regime for η between 45 and 80 here.

decrease abruptly and for even larger values of η, decrease slower, as η^{-3}. The noise in the quantized region and around it seems to be well approximated by $Q(e-Q)$, reminiscent of the noise for fermions in narrow quantum wires. However, this does not last when η becomes noticeably out of the region of almost quantized charge, since Q and Q_0 behave as η^{-3}, whereas S and S_0 decay only as η^{-2}.

Figure 5 shows noise and charge with and without interactions versus η for $k_F a=0.1$. Figure 6 shows a comparison between S and a least-squares fit of the form $CQ(e-Q)$, in the quantized charge regime, where C is the only adjustable parameter (see captions for details). We now turn to analytical justifications of the previous assertions.

We now explain why pumping is impossible for $k_F a=n\pi/2$. Since $\sin(2k_F a)=0$, the scattering matrix now depends only on a single parameter, the combination $(\bar{X}+\bar{Y})$ [see Eqs. (18)–(20)], so we denote by $s_{ij}^{(0)\prime}$ its derivative with respect to $\bar{X}+\bar{Y}$. Thus,

$$Q_0=\frac{e}{\pi}\int_0^{2\pi/\omega}\mathrm{Im}\left[\left(\sum_{j=1}^{2}s_{1j}^{(0)\prime}(\bar{X}+\bar{Y})s_{1j}^{(0)*}(\bar{X}+\bar{Y})\right)\right]\times\frac{d}{dt}(\bar{X}+\bar{Y})dt=0 \tag{34}$$

because the bracket is just $(1/2)d/d(\bar{X}+\bar{Y})(|s_{11}^{(0)}|^2+|s_{12}^{(0)}|^2)$. Then, we look at the case when $k_F a$ is close to $n\pi/2$ but different from it. Clearly, when $\sin(2k_F a)$ is small, for $\eta|\sin(k_F a)|<1$, we will be back to the former case, so Q_0 can start to level noticeably from 0 only for η values larger than a critical value η_{c1}, which is proportional to $1/|\sin(2k_F a)|$, independent of X_0. There is at least another scale, namely, X_0. For X_0 large (typically larger than 10) and η smaller than X_0, the pumping contour is a circle which does not enclose the origin and both transmission $s_{12}^{(0)}$ and derivatives of the transmission coefficients $\partial s_{12}^{(0)}/\partial X$ are small. Q_0 will remain very small. Thus, to have a significant Q_0, one needs η

$>\max[X_0,1/|\sin(2k_Fa)|]$, where $\max(x,y)$ is the larger of x and y. For even larger η, when terms such as $\sin(2k_Fa)\bar{X}\bar{Y}$ dominate over terms linear in $\bar{X}$ or $\bar{Y}$, i.e., $[|\sin(2k_Fa)|\eta] \gg 1$, it is possible to expand in η^{-1} and we are back to the large pumping regime where Q decays as η^{-3}. So, for $|\sin(2k_Fa)|$ smaller than X_0, there will be a region between $\max[X_0,C_1/\sin(2k_Fa)]$ and $C_2/\sin(2k_Fa)$, where C_1 and C_2 are constants, where Q_0 is appreciable. These very qualitative arguments however do not prove that the charge is almost quantized in this interval, whose width is of the order $|\sin(2k_Fa)|^{-1}$.

We now turn to the analytical explanation of the $Q(e-Q)$ behavior in the quantized charge regime. It is useful to disentangle the effects of the fluctuations of T and those of the phase θ. We start from Eq. (29) and use the fact that for any periodic function $f(x)$, if $\hat{f}_n$ denotes the Fourier component at frequency $n\omega$, denoting $x=\omega t$,

$$\sum_{n\geq 1} n|\hat{f}_n|^2 = \int_0^{2\pi}\int_0^{2\pi} \frac{|f(x)-f(x')|}{(x-x')^2}\frac{dxdx'}{4\pi^2}. \tag{35}$$

Applying the former equality to $\hat{M}_{1,1n}$, $\hat{M}_{1,2n}$, $\hat{M}_{2,1n}$, and $\hat{M}_{2,2n}$ and denoting by $M_{11}(x)$ the inverse Fourier transform of $\hat{M}_{1,1n}$, we get

$$\begin{aligned} S = e^2\frac{\omega}{2\pi}\int_0^{2\pi}\int_0^{2\pi} &(|M_{11}(x)-M_{11}(x')|^2 + |M_{12}(x)-M_{12}(x')|^2 \\ &+ |M_{21}(x)-M_{12}(x')|^2 + |M_{22}(x)-M_{22}(x')|^2) \\ &\times (x-x')^{-2}\frac{dxdx'}{4\pi^2}. \end{aligned} \tag{36}$$

In terms of θ and T, using Eqs. (4) and (24), this reads

$$S = 8e^2\left(\frac{\omega}{2\pi}\right)(I_1+I_2), \tag{37}$$

with

$$I_1 = \int_0^{2\pi}\int_0^{2\pi}\frac{|T(x)-T(x')|^2}{(x-x')^2}\frac{dxdx'}{4\pi^2}, \tag{38}$$

$$I_2 = \int_0^{2\pi}\int_0^{2\pi}\frac{|g(x)-g(x')|^2}{(x-x')^2}\frac{dxdx'}{4\pi^2}, \tag{39}$$

with $g(x)=\sqrt{T(1-T)}e^{i\theta}$. We expand $|g(x)-g(x')|^2$ and rewrite I_2 in the form

$$I_2 = J_1 + J_2, \tag{40}$$

with

$$J_1 = \int_0^{2\pi}\int_0^{2\pi}\left\{\frac{\sqrt{T(x)[1-T(x)]}-\sqrt{T(x')[1-T(x')]}}{(x-x')}\right\}^2 \times\frac{dxdx'}{4\pi^2}, \tag{41}$$

$$J_2 = \int_0^{2\pi}\int_0^{2\pi}\sqrt{T(x)(1-T(x))}\sqrt{T(x')(1-T(x'))} \times\frac{|e^{-i\theta(x)}-e^{-i\theta(x')}|^2}{(x-x')^2}\frac{dxdx'}{4\pi^2}. \tag{42}$$

Thus, noise breaks into three parts, one related to the fluctuations of T, another to the fluctuations of $\sqrt{T(1-T)}$, and the last to the fluctuations of θ, in fact, to the variations of the slope $d\theta/dx$, since θ has to vary by 2π in one cycle to get appreciable pumped charge. The physical message is that if the fluctuations of T are much smaller than the fluctuations of the phase, then noise shows a $Q(e-Q)$ behavior. Otherwise, fluctuations of T bring an extra noise that does not contribute to the pumped charge and overall noise is thus larger than $Q(e-Q)$. We now try to establish this more firmly.

In this paragraph, we now show that, for $Q_0 \leq e/2$,

$$S_0 \geq 8\left(\frac{\omega}{2\pi}\right)Q_0(e-Q_0). \tag{43}$$

The first integral J_1 involves solely the fluctuations of T and can be rewritten as

$$J_1 = \sum_{n=1}^{\infty} n|\widehat{\sqrt{T(1-T)}}_n|^2. \tag{44}$$

We have a lower bound for J_1 by replacing n by 1 in all terms of the sum except the term for $n=0$,

$$J_1 \geq \sum_{n=0}^{\infty}|\widehat{\sqrt{T(1-T)}}_n|^2 - |\widehat{\sqrt{T(1-T)}}_0|^2. \tag{45}$$

Using then the Parseval identity for the first term and using the notation $\langle f\rangle \equiv \int_0^{2\pi} f(x)dx/2\pi$ for the second,

$$\begin{aligned} J_1 &\geq \langle|\sqrt{T(1-T)}|^2\rangle - \langle\sqrt{T(1-T)}\rangle^2 \\ &= \langle T\rangle - \langle T^2\rangle - \langle\sqrt{T(1-T)}\rangle^2. \end{aligned} \tag{46}$$

Now for J_2, applying twice the Hölder inequality,

$$\begin{aligned} J_2 &= \int_0^{2\pi}\frac{dx'}{2\pi}\sqrt{T(x')[1-T(x')]}\int_0^{2\pi}\sqrt{T(x)[1-T(x)]} \\ &\quad\times\frac{|e^{-i\theta(x)}-e^{-i\theta(x')}|^2}{(x-x')^2}\frac{dx}{2\pi} \\ &\geq \int_0^{2\pi}\frac{dx'}{2\pi}\sqrt{T(x')[1-T(x')]}\langle\sqrt{T(1-T)}\rangle \\ &\quad\times\int_0^{2\pi}\frac{|e^{-i\theta(x)}-e^{-i\theta(x')}|^2}{(x-x')^2}\frac{dx}{2\pi} \\ &\geq \langle\sqrt{T(1-T)}\rangle^2\int_0^{2\pi}\frac{dx}{2\pi}\int_0^{2\pi}\frac{dx'}{2\pi}\frac{|e^{-i\theta(x)}-e^{-i\theta(x')}|^2}{(x-x')^2}. \end{aligned} \tag{47}$$

For the last double integral, we proceed as before, going again to Fourier transform, isolating the component of order 0, and using the Parseval identity, it is larger than $\langle|e^{i\theta}|^2\rangle - |\langle e^{i\theta}\rangle|^2$. Always $|e^{i\theta}|=1$ and, for reasonable $\theta(x)$, we can assume symmetry x into $-x$ which implies $\langle\sin\theta\rangle=0$. We can also assume symmetry when x is changed into $\pi-x$, which implies $\langle\cos\theta\rangle=0$. We have

$$J_2 \geq \langle\sqrt{T(1-T)}\rangle^2. \tag{48}$$

Now, for I_1, using the same technique (going to Fourier transform and isolating the $n=0$ component),

$$I_1 \geq \langle T^2\rangle - \langle T\rangle^2. \tag{49}$$

Putting everything together

$$I_1 + I_2 \geq \langle T^2\rangle - \langle T\rangle^2 + \langle T\rangle - \langle T^2\rangle - \langle\sqrt{T(1-T)}\rangle^2 + \langle\sqrt{T(1-T)}\rangle^2 = \langle T\rangle - \langle T\rangle^2. \tag{50}$$

Now, assuming that $\int_0^{2\pi}\frac{d\phi}{dx}\frac{dx}{2\pi}=0$ (the circulation of ϕ is zero in one cycle),

$$Q = e\left(1 - \left\langle T\frac{d\theta}{dx}\right\rangle\right), \tag{51}$$

which implies

$$Q(e-Q) = e^2\left(\left\langle T\frac{d\theta}{dx}\right\rangle - \left\langle T\frac{d\theta}{dx}\right\rangle^2\right). \tag{52}$$

Moreover, by the Hölder inequality,

$$\left\langle T\frac{d\theta}{dx}\right\rangle \geq \langle T\rangle\left\langle\frac{d\theta}{dx}\right\rangle = \langle T\rangle. \tag{53}$$

Now, for any $y \geq 1/2$, $y(1-y)$ is a decreasing function of y. Thus, if $\langle T\frac{d\theta}{dx}\rangle \geq 1/2$, i.e., $Q \leq e/2$, then

$$\langle T\rangle - \langle T\rangle^2 \geq \left\langle T\frac{d\theta}{dx}\right\rangle - \left\langle T\frac{d\theta}{dx}\right\rangle^2. \tag{54}$$

Thus $S \geq 8(\frac{\omega}{2\pi})Q_0(e-Q_0)$, which is Eq. (43).

Now we turn to the case of interest, when $Q_0 \geq e/2$, for example, in the quantized region. We were not able to provide a general analytical proof of $Q_0(e-Q_0)$ behavior. We first look at simple limiting cases. Then, we examine the particular case of the two-delta-potential model.

First, in a situation where $T(x)$ is a constant T (with T small to have almost charge quantization), then $Q_0=e(1-T)$ and $S_0=8C_0(\frac{\omega}{2\pi})Q_0(e-Q_0)$, with C_0 depending on the shape of $\theta(x)$, but always C_0 is greater than 1. In the case of constant slope $d\theta/dx=1$, $C_0=1$. Second, in the situation where $d\theta/dx$ is constant but $T(x)$ arbitrary, we have $\langle T(d\theta/dx)\rangle=\langle T\rangle$ and thus again Eq. (43) holds for any Q_0, not just for Q_0 smaller than $e/2$.

However, in practice, θ varies abruptly by π in the vicinity of resonances. This is different from optimal pumping strategies which have been studied before.[15,19] We first give qualitative arguments and then give precise calculations for the model studied here. Let us look at the contribution of J_2 [Eq. (48)] to the noise. When x and x' are both close to resonances $|e^{-i\theta(x)}-e^{-i\theta(x')}|^2/(x-x')^2$ behaves as $(d\theta/dx)^2$ for $(x-x')(d\theta/dx)<1$. If meanwhile, $T(x)$ does not vary too much and assumes the value T_i, the contribution of this region in the plane (x,x') to J_2 will be $T_i(1-T_i)/4$. The 1/4 comes from the fact that θ varies suddenly only by π and not by 2π at each resonance. Another contribution will come from the regions where x is within the resonance but x' just outside or vice versa. Let us take x' outside to be precise. Then $|e^{-i\theta(x)}-e^{-i\theta(x')}|=2$ and integration on x and x' will give a contribution mainly from x' just outside due to the rapidly decreasing factor $(x-x')^{-2}$. This will eventually give another factor $T_i(1-T_i)/4$. Despite the nonlocal character of the integrand in J_2, regions where x and x' are both far from a resonance make very little contribution to J_2. For Q, one gets $Q=e(1-\Sigma_i T_i/2)$. In the simple case where there are only two resonances and the T_i's are equal, then $Q=e(1-T_1)$ and $J_2=T_1(1-T_1)$. If I_1 and J_1, which are related to the fluctuations of $T(x)$, are much smaller than J_2, then this leads to $S_0=8(\omega/2\pi)Q_0(e-Q_0)$.

We now test the former very qualitative ideas by analytical calculations on our particular model. A first thing to be noted is that the phase ϕ does not intervene in the noise, which is normal since noise is related to the two-particle scattering matrix. On the contrary, it does formally enter the equation for the pumped charge [see Eqs. (4), (15), and (16)]. Nevertheless, the variation of ϕ when ωt varies in one period is always zero so that ϕ does not play any role. This is due to the fact that ϕ is the phase of D [see Eqs. (19) and (20)]. If the real part of D becomes negative, then its imaginary part cannot be zero and thus ϕ can never be equal to $-\pi$ or π. Moreover, ϕ varies by strictly less than 2π during one period and thus its circulation is zero, giving no contribution to Q and Q_0.

We now turn to the variations of θ. Charge quantization necessitates η larger than $\sqrt{2}X_0$ and k_Fa small. We thus approximate $\cos(2k_Fa)$ by 1 and set $u=\sin(2k_Fa)\ll 1$. θ can be written as

$$\theta = \arg(n), \tag{55}$$

with

$$n = \bar{X}\bar{Y}u + \bar{X} + \bar{Y} + iu(\bar{Y}-\bar{X}). \tag{56}$$

There exist two values of η, η_1, and η_2, which play a particular role. For $\eta_1 \leq \eta \leq \eta_2$, when ωt varies by 2π, $\mathrm{Re}(n)$ changes twice its sign and θ varies by 2π. In the model studied here, $\eta_1=X_0\sqrt{2}$ and $\eta_2=X_0\sqrt{2}+2\sqrt{2}u^{-1}$. Outside this interval of η, the increase in θ when ωt varies by 2π is zero, not 2π. The reason is the following. For $\eta \leq \eta_1$, $\mathrm{Re}(n)$ is always positive and the phase θ remains confined in an interval contained in $[-\frac{\pi}{2},\frac{\pi}{2}]$. For $\eta > \eta_2$, $\mathrm{Re}(n)$ changes four times its sign but the contour described by n, in the complex plane, as ωt varies by 2π, does not enclose the origin. It can be seen directly for it is impossible to have $\mathrm{Im}(n)=0$ and $\mathrm{Re}(n)\leq 0$ at the same time. Thus, it is not surprising that the quantized region corresponds approximately to the interval $[\eta_1,\eta_2]$.

We now examine more precisely the variations of θ and T. Apart from a small variation around $x\equiv\omega t=-3\pi/4$, θ, as a function of x, is essentially flat, except around two points x_1

and x_2. In practice, x_1 is close to $-\pi/4$ and x_2 close to $3\pi/4$. Around those two points, θ increases fastly by almost π each time. We can thus model the function θ by

$$\theta = -\pi \quad \text{for} \quad x \le x_1 - l,$$

$$\theta = \frac{\pi}{2}\left(\frac{x-x_1}{l} - 1\right) \quad \text{for} \quad x_1 - l \le x \le x_1 + l,$$

$$\theta = 0, \quad \text{for} \quad x_1 + l \le x \le x_2 - l$$

$$\theta = \frac{\pi}{2}\left(1 + \frac{x-x_2}{l}\right) \quad \text{for} \quad x_2 - l \le x \le x_2 + l,$$

$$\theta = \pi, \quad \text{for} \quad x \ge x_2 + l \tag{57}$$

with l being a small distance. As for T, $T(x)$ shows a large peak around $x=x_s=-3\pi/4$, and two smaller peaks, practically identical, centered around x_1 and x_2. Away from these values of x, $T(x)$ is almost zero.

Now, we look at the implications for the charge and noise. When calculating the pumped charge without interaction via the formula $\frac{1}{2\pi}\int_0^{2\pi}\frac{d\theta}{dx}[1-T(x)]dx$, the region around $x=x_s$ does not bring much contribution because variations of θ are small here. For the calculation of Q_0 and S_0, we can ignore the large peak in T and thus model $T(x)$ by

$$T(x) = T_1 \exp\left(\frac{(x-x_i)^2}{(x-x_i)^2 - l^2}\right) \quad \text{for} \quad |x-x_i| \le l,$$

$$T(x) = 0, \quad \text{otherwise}. \tag{58}$$

T has to be derivable in order to avoid logarithmic divergences due to the factor $1/(x-x')^2$ in Eqs. (38) and (39). Then, it is possible to perform analytical calculations which give

$$Q_0 = e(1 - C_1 T_1), \tag{59}$$

$$S_0 = C_2\left(\frac{\omega}{2\pi}\right)(e - Q_0)[e(1 - C_3) + C_3 Q_0)]. \tag{60}$$

Detailed calculations involving the integrals I_1 and I_2 and the constants C_1, C_2, and C_3 are given in Appendix B. C_3 is smaller than 1 (approximately 0.58). Even if it is not exactly of the form $Q_0(e-Q_0)$, when there is good charge quantization, i.e., when Q_0 is not far from 1, S_0 goes as $(e-Q_0)$. The essential thing is that T and θ vary rapidly around certain values of ωt. We do not get exactly $Q_0(e-Q_0)$ because T varies substantially when θ jumps. One might wonder if the results here are particular to our model. In fact, brisk variations of the phase are widely shared by many types of models.[17,18]

The third case corresponds to $k_F a$ not close to $n\pi/2$. In this case, for large X_0, Q is almost zero except in the vicinity of a value η_c, which is very near $\sqrt{2}X_0$; numerically it seems that η_c is always a little less than this value. The maximum pumped charge is of order e but no longer close to one-electron charge. Noise has a double peak structure around η_c. A rough qualitative picture of this can be seen in Eq. (19) because, as soon as the integration contour does not get close to the point $X(t)=Y(t)=0$, for any t, the integrands in Eqs. (15) and (16) are very small. An example is shown in Fig. 7.

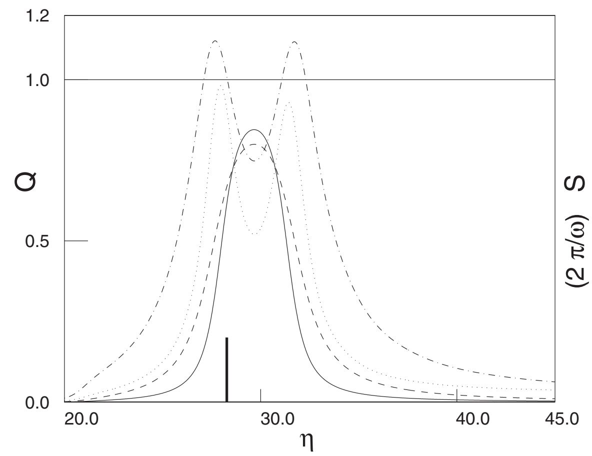

FIG. 7. Same as Fig. 5 except that $k_F a = 0.4$. The solid horizontal line of ordinate 1 and the thick vertical line at abscissa $X_0\sqrt{2}$ are just guides for the eyes.

IV. CONCLUSION

We have studied the influence of weak electron-electron interactions on pumped charge and noise in the adiabatic regime in a mesoscopic one-dimensional disordered wire. Within the two-delta-potential barrier model, analytical results were obtained for the charge and noise. Results were analyzed numerically for local pumping fields with a harmonic dependence. Without any voltage offset, at weak pumping amplitudes, interactions tend to enhance the pumped charge, as $l^{-2\alpha}$, where l is the interaction parameter. For fairly large pumping amplitudes, it is exactly the reverse; Q and Q_0 both decrease as η^{-3}, but Q remains smaller than Q_0 by a factor $l^{2\alpha}$. At very large pumping amplitudes, Q and Q_0 are practically the same. As to the pumping noise, at weak amplitudes, it increases with interactions, but in the same way as the charge, so that the Fano factor remains constant, independent of the interactions. For moderate pumping amplitudes, noise has a double peak structure around the maximum of pumped charge. For large amplitudes, the noise decreases slower than the charge, as η^{-2}, and for very large amplitudes, noise with and without interactions become approximately the same.

As emphasized in Ref. 9, interactions tend to make resonances sharper, which is conducive to obtaining an almost quantized pumped charge. However, it is not sufficient to enclose a resonance; it is also necessary that the pumping contour does not go too far from the resonance. Otherwise, the noise is appreciable and the signal Q can even be very small.

In the case of constant offset X_0, the behavior depends if we are close to a resonance $k_F a = n\pi/2$ in the two-delta-potential model. Close to a resonance, there is a region of almost quantized pumped charge where the noise seems to follow a $Q(e-Q)$ behavior, reminiscent of the noise reduction in quantum wires for good transmission by a $T(1-T)$ factor, where T is the modulus of the energy transmission coefficient. Quite generally, noise breaks up into pieces due to fluctuations of T and those due to fluctuations of θ. We believe that in the quantized region, the fluctuations of θ are predominant and give rise to the $Q(e-Q)$ behavior.

Interactions do help in having a charge closer to e and to reduce the noise. However, it does not change the range of pumping amplitudes, where quantized charge is observed, i.e., the width of the quantized region practically does not depend on the interactions. Qualitative arguments seem to indicate that this width scales as $\sin(2k_Fa)^{-1}$ close to $k_Fa = n\pi/2$. Further from the resonance, the maximum charge which can be pumped becomes of order but less than e. Moreover, the region of quantized charge shrinks to a narrow window of pumping amplitudes around a value close to $X_0\sqrt{2}$.

In summary, our study of noise shows that interactions tend to increase the quality of pumping. However, two conditions need to be met: first, to operate at certain wave vectors favoring sharp resonances and second to have a pumping contour which encircles the resonant point, passing not very far from it. Otherwise, only noise is produced and the quantization of the charge is not achieved. These restrictions were not pointed out in previous works. In addition, in the quantized charge region, noise vanishes as $e-Q$.

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APPENDIX A

In this appendix, we consider the limit of large pumping. In order to explain why, for offset $X_0=0$, Q_0 behaves like η^{-3} at large η, we use Eqs. (15) and (18)–(20). Let us look first at the terms involving $d\bar{X}/dt$.

$\bar{X}$, $\bar{Y}$, as well as their time derivatives will be of order η, except at isolated particular points. The term $|\partial \ln s_{12}^{(0)}/\partial\bar{X}|^2$ can be expanded in powers of $1/\bar{X}$ and $1/\bar{Y}$,

$$\left|\frac{\partial \ln s_{12}^{(0)}}{\partial \bar{X}}\right|^2 = \frac{1}{\bar{X}^2} - \frac{\cot(2k_Fa)}{\bar{X}^3} + \frac{3\cot^2(2k_Fa)-1}{4\bar{X}^4} - \frac{1+\cot^2(2k_Fa)}{4\bar{X}^2\bar{Y}^2} + \frac{\cot^2(2k_Fa)-1}{2\bar{X}^3\bar{Y}} + o(\eta^{-5}). \tag{A1}$$

All terms multiplied by $d\bar{X}/dt$ and then integrated over one period give zero. Note that a term such as $1/(\bar{X}^4\bar{Y})$, which is in η^{-5}, would not give 0. Thus, for large η, the term proportional to $d\bar{X}/dt$ in Q behaves at least as η^{-3}.

For the term involving $d\bar{Y}/dt$, the situation is simpler. $d\bar{Y}/dt$ goes as η, but since D goes as η^2, T_0 goes as η^{-4}; thus this term is at least in η^{-3}. Now the remainder of contributions to $Q-Q_0$ behaves at least like η^{-4} for large η; it can be seen from Eq. (16). For large η, since T_0 goes as η^{-4}, the quantity $T_0/[1+T_0(A^{2\alpha}-1)]$ is practically equivalent to $T_0 \sim \eta^{-4}$. $\mathrm{Im}\{\partial \ln s_{12}^{(0)}/\partial\bar{X}\}$ goes as η^{-1} for large η. The same holds for $\mathrm{Im}\{\partial \ln s_{12}^{(0)}/\partial\bar{Y}\}$. In the integral in the right-hand side of Eq. (16), $\mathrm{Im}\{\partial \ln s_{12}^{(0)}/\partial\bar{X}\}$ is larger, by η, than $|(\partial \ln s_{12}^{(0)}/\partial\bar{X})|^2$. Then, $\mathrm{Im}\{\partial \ln s_{12}^{(0)}/\partial\bar{X}\}$ is of order η^{-1} whereas T_0 is of order η^{-4}. Finally, the integrand is at least of order $\mathrm{Im}\{\partial \ln s_{12}^{(0)}/\partial\bar{X}\}(d\bar{X}/dt)T_0$orIm$\{\partial \ln s_{12}^{(0)}/\partial\bar{Y}\}(d\bar{Y}/dt)T_0$, which are both at least of order $\eta^{-1}\times\eta\times\eta^{-4}\sim\eta^{-4}$.

We now evaluate the behavior of the noise for large η. When $\omega \ll \epsilon_F$, only the low order Fourier components of $\eta(t)$ are important. ϵ_1 and ϵ_2' will be much smaller than ϵ_F. At $\epsilon_1=\epsilon_2'=0$, $s_{11}^{(0)} \simeq -e^{-2ik_Fa}[1+i/\bar{X}+o(\eta^{-2})]$ and $s_{22}^{(0)}$ is the same but $\bar{X}$ is replaced by $\bar{Y}$. $s_{12}^{(0)}=-(1/2\bar{X}\bar{Y})[1+i\cot(2k_Fa)] + o(\eta^{-3})$. As a result, in Eq. (23), $s(\epsilon_1,t)^\dagger\sigma_z s(\epsilon_2,t)=\sigma_z + o(\eta^{-2})$, the same holds for $s(\epsilon_2,t')^\dagger\sigma_z s(\epsilon_1,t')$, so that the trace is $o(\eta^{-2})$, which yields S of order η^{-2} at least.

APPENDIX B

In this appendix, we show the result of calculations for the integrals I_1 and I_2 using Eqs. (57) and (58) for $\theta(x)$ and $T(x)$. The integrand of I_1 and I_2 are nonzero only if at least one of x or x' is within distance l from an x_i, $i=1,2$. We shall need the integrals

$$M_0 = \int_0^1 \exp\left(\frac{x^2}{x^2-1}\right)dx \simeq 0.603, \tag{B1}$$

$$K_1 = \int_{-1}^1 \exp\left(\frac{x^2}{x^2-1}\right)(1-x)^{-1}dx \simeq 1.207, \tag{B2}$$

$$K_2 = \int_{-1}^1\int_{-1}^1 \left[\exp\left(\frac{x^2}{x^2-1}\right) - \exp\left(\frac{y^2}{y^2-1}\right)\right]^2 (x-y)^{-2}dxdy \simeq 2.088, \tag{B3}$$

$$L_1 = \int_{-1}^1\int_{-1}^1 \left\{\exp\left(\frac{x^2}{x^2-1}\right) + \exp\left(\frac{y^2}{y^2-1}\right) - 2\exp\left[\frac{1}{2}\left(\frac{x^2}{x^2-1}+\frac{y^2}{y^2-1}\right)\right]\cos\left[\frac{\pi}{2}(x-y)\right]\right\} \times (x-y)^{-2}dxdy \simeq 7.997, \tag{B4}$$

$$L_2 = \int_{-1}^1\int_{-1}^1 \left\{\exp\left(\frac{2x^2}{x^2-1}\right) + \exp\left(\frac{2y^2}{y^2-1}\right) - \exp\left[\frac{1}{2}\left(\frac{x^2}{x^2-1}+\frac{y^2}{y^2-1}\right)\right] \times \left[\exp\left(\frac{x^2}{x^2-1}\right) + \exp\left(\frac{y^2}{y^2-1}\right)\right]\right.$$

$$\times\cos\left[\frac{\pi}{2}(x-y)\right]\Bigg\}(x-y)^{-2}dxdy \simeq 6.591. \quad (B5)$$

Then, plugging these values into Eqs. (15), (38), and (39),

$$Q_0 = e(1 - T_1 C_1), \quad (B6)$$

$$S_0 = C_2\left(\frac{\omega}{2\pi}\right)(e - Q_0)[e - C_3(e - Q_0)], \quad (B7)$$

with

$$C_1 = M_0 \simeq 0.603, \quad (B8)$$

$$C_2 = \frac{2}{\pi^2}(8K_1 + 2L_1)/M_0 \simeq 8.613, \quad (B9)$$

$$C_3 = \frac{L_2 - K_2}{(L_1 + 4K_1)M_0} \simeq 0.58. \quad (B10)$$

APPENDIX C

In this appendix, we show the equivalence of Eq. (29) with Eqs. (24a)–(24c) of Ref. 16. There, the noise power $P_{\alpha\beta}$ between leads α and β was given by

$$P_{\alpha\beta} = 2\frac{e^2}{h}\sum_{q=1}^{\infty} q\hbar\omega C^{\rm sym}_{\alpha\beta,q}(\mu), \quad (C1)$$

$$C^{\rm sym}_{\alpha\beta,q}(E) = \frac{C_{\alpha\beta,q}(E) + C_{\alpha\beta,-q}(E)}{2}, \quad (C2)$$

$$C_{\alpha\beta,q}(E) = \sum_{\gamma}\sum_{\delta}[s^*_{\alpha\gamma}(E)s_{\alpha\delta}(E)]_q[s^*_{\beta\delta}(E)s_{\beta\gamma}(E)]_{-q}, \quad (C3)$$

where $[A]_q$ denotes the Fourier transform at frequency $q\omega$ of a time-dependent quantity A. In our case, there are only two leads so that indices γ and δ are either 1 or 2. We are interested in P_{11}, so we make $\alpha = \beta = 1$. Then, inserting the value of the scattering matrix elements according to Eq. (4) leads to

$$P_{11} = 4e^2\left(\frac{\omega}{2\pi}\right)\left\{\sum_{q\geq 1} q\left[|T_q|^2 + \frac{1}{2}[|(\sqrt{RT}e^{i\theta})_q|^2 + |(\sqrt{RT}e^{-i\theta})_q|^2]\right]\right\}. \quad (C4)$$

On the other hand, using Eq. (29), we obtain

$$S = 8e^2\left(\frac{\omega}{2\pi}\right)\left\{\sum_{n\geq 1} n\left[|T_n|^2 + \frac{1}{2}[|(\sqrt{RT}e^{i\theta})_n|^2 + |(\sqrt{RT}e^{-i\theta})_n|^2]\right]\right\}, \quad (C5)$$

which is, apart from an overall factor 2, the same as Eq. (C4), with q changed to n.

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